

Table 12.20 *Product Array Design for Chemical Process Experiment*

			<i>E</i>	-	+	-	+
			<i>D</i>	-	-	+	+
<i>A</i>	<i>B</i>	<i>C</i>					
-	-	0	37.29	57.81	42.87	47.07	
+	-	0	4.35	24.89	8.23	14.69	
-	+	0	9.51	13.21	10.10	11.19	
+	+	0	9.15	13.39	10.30	11.23	
-	0	-	20.24	27.71	22.28	24.32	
+	0	-	4.48	11.40	5.44	8.23	
-	0	+	18.40	30.65	20.24	24.45	
+	0	+	2.29	14.94	4.30	8.49	
0	-	-	22.42	42.68	21.64	30.30	
0	+	-	10.08	13.56	9.85	11.38	
0	-	+	13.19	50.60	18.84	30.97	
0	+	+	7.44	15.21	9.78	11.82	
0	0	0	12.29	19.62	13.14	14.54	
0	0	0	11.49	20.60	12.06	13.49	
0	0	0	12.20	20.15	14.06	13.38	

4. Because the elastometric connector experiments described in Section 12.3 required physical experimentation, Song and Lawson (1988) suggested using a single array design to save on the number of experiments required. Table 12.21 shows a resolution IV 2^{7-2} fractional factorial design and the resulting pull-off force. The factor names and levels are the same as those shown in Table 12.3 of Section 12.3. The generators for the design were $F = ABC$ and $G = ABD$. The data in Table 12.21 is in the data frame `connector` in the `daewr` package.

- What is the defining relation for this design, and how many of the 12 control-by-noise factor interactions can be estimated clear of other main effects or two-factor interactions?
- Using the `FrF2` package, can you find a 32-run resolution IV design that has more control-by-noise factor interactions clear? If so, what are the generators?
- Using the data in Table 12.21, calculate a set of 31 saturated effects for this design, and make a normal plot to determine which effects, interactions, and confounded strings of two-factor interactions appear to be significant. Is there a clear interpretation of any confounded strings of interactions?
- Make interaction and contour plots of any two-factor interactions between control factors and noise factors and use them to choose the level of control factors that will minimize the effect of noise factors.

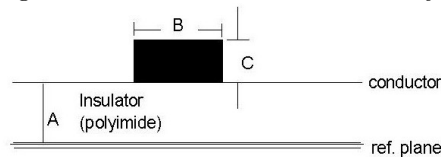
- (e) Are there any adjustment factors? If so, what levels should be chosen to maximize the pull-off force?

Table 12.21 *Single-Array Experiment for Elastometric Connector*

<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>	<i>G</i>	Pull-Off Force
Low	High	High	Low	120	150	75	22.0
High	High	High	High	120	150	75	27.6
High	High	High	Low	120	72	25	22.1
High	High	Low	High	120	72	75	20.2
Low	Low	Low	High	120	72	75	18.9
High	High	High	High	24	150	25	13.8
Low	High	High	High	24	72	75	15.2
Low	Low	Low	Low	120	150	25	23.1
High	Low	High	High	24	72	75	16.1
High	Low	High	High	120	72	25	20.1
Low	Low	High	High	120	150	75	21.9
High	Low	Low	Low	24	72	25	17.1
High	Low	High	Low	24	150	25	18.1
Low	High	Low	High	24	150	75	9.6
Low	Low	High	Low	24	72	75	18.3
Low	High	Low	Low	120	72	75	19.1
High	Low	High	Low	120	150	75	27.0
Low	Low	Low	Low	24	150	75	17.3
Low	Low	High	High	24	150	25	17.7
Low	Low	Low	High	24	72	25	14.7
High	Low	Low	Low	120	72	75	22.2
Low	High	High	Low	24	150	25	21.1
Low	Low	High	Low	120	72	25	24.3
Low	High	Low	Low	24	72	25	13.9
High	Low	Low	High	24	150	75	19.4
High	Low	Low	High	120	150	25	22.7
High	High	Low	High	24	72	25	14.6
High	High	Low	Low	120	150	25	23.3
High	High	High	Low	24	72	75	20.4
Low	High	Low	High	120	150	25	22.6
Low	High	High	High	120	72	25	21.0
High	High	Low	Low	24	150	75	17.5

5. Reconsider the data from the injection molding experiment in Table 12.13.
- Fit the model $y = \beta_0 + \beta_A X_A + \beta_B X_B + \beta_{A \times B} X_A X_B$ in the adjustment factors using the method of least squares.
 - Simultaneously estimate the variance of the residuals at each level of the dispersion factor, C , and compute the weighted least squares estimates of the coefficients in the model in (a) using the `lme` function in the `nlme` package as shown in Section 12.6.1, page 543.
6. In the design of a thin film redistribution layer (cited by J. Lorenzen, IBM Kingston and discussed by Lawson and Madrigal (1994)), the circuit impedance (Z) is a function of three design factors: the A , insulator thickness; B , linewidth; and C , line height as shown in Figure 12.21. From engineering first principles, it can be shown that the impedance is given by Equation (12.10), where ϵ is the dielectric constant of the insulator and is assumed to be constant at 3.10.

$$Z = f(A, B, C) = \frac{87.0}{\sqrt{\epsilon + 1.41}} \ln \left(\frac{5.98A}{0.8B + C} \right) \quad (12.10)$$

Figure 12.21 *Thin Film Distribution Layer*

The nominal or mean values of A , B , and C can be specified by the design engineer, but in actual circuits these characteristics will vary from their nominal values due to manufacturing imperfections and wear during use. The table below shows the feasible range and tolerance limits for these variables.

Control Factor	Feasible Range (μm)	Low-Cost Tolerance Range (μm)
A : Insulator thickness	20-30	± 1.0
B : Linewidth	12.5-17.5	± 0.67
C : Line height	4-6	± 0.33

The goal is to find the nominal settings of A , B , and C that will result in impedance = 85, with minimum variance.

- (a) Construct a control-factor array in the design factors A, B, and C using a 2^3 factorial design with the ends of the feasible range as low and high values of the factors.
 - (b) Construct a 3^3 noise-factor array (in deviations from the nominal settings in the control-factor array) using the 3 run table of Wang, Fang, and Lin (i.e., $\mathbf{nA} \leftarrow \mathbf{c}(\mathbf{c}(.1667, .5, .8333))$) etc. Translate the levels in Table 12.16 into values in the low cost tolerance range of each factor using the formulas: $\mathbf{dA} \leftarrow 1 * (\mathbf{nA} - .5) / .5$, $\mathbf{dB} \leftarrow .667 * ((\mathbf{nB} - .5) / .5)$, and $\mathbf{dC} \leftarrow .333 * ((\mathbf{nC} - .5) / .5)$.
 - (c) Create a product array of 216 rows by merging the noise factor array with each run or combination of factor levels in the control factor array.
 - (d) Evaluate Equation (12.10) for each run or combination of levels in the product array, substituting for A, A+dA. Substituting for B, B+dB, and substituting for C, C+dC to simulate the impedance of an actual manufactured thin film redistribution layer.
 - (e) Calculate the mean impedance and variance of impedance across the 27 runs in the noise array for each run or combination of factor levels in the control array. Include the mean impedance and $\log(\text{variance})$ of impedance as new variables in the control array.
 - (f) Determine which control factors have significant effects on the mean impedance and $\log(\text{variance})$ of impedance.
7. Consider the data for the product array design for the elastometric connector shown in Figure 12.4 (the data is in the data frame `prodstd` in the `daewr` package).
 - (a) Calculate the mean pull-off force and log variance of the pull-off force across the noise array for each run in the control-factor array.
 - (b) Analyze the data using the location-dispersion modeling method.
 - (c) Do you find the same optimal levels of the control factors as identified in Section 12.4.2?
 - (d) Is any information lost when analyzing this data using location-dispersion modeling?
 8. An experiment originally performed by the National Railway Corporation of Japan (Taguchi and Wu, 1980) was reanalyzed by Box and Meyer (1986b). The control factors in the design were *A*, kind of welding rods; *B*, period of drying; *C*, welded materials; *D*, thickness; *E*, angle; *F*, opening; *G*, current; *H*, welding method; and *J*, preheating. Taguchi and Wu also considered the interactions *AC*, *AG*, *AH*, and *GH*. The design and response $y =$ tensile strength of welds is shown in Table 12.22 where e_1 and e_2 are unassigned columns that represent confounded interactions. The data is also in the data frame `WeldS` in the `daewr` package.
 - (a) Calculate effects for each of the 15 columns in Table 12.24, and make a normal plot to identify factors that affect the tensile strength.

Table 12.22 *Design and Tensile Strength for Weld Experiment*

<i>D</i>	<i>H</i>	<i>G</i>	<i>A</i>	<i>F</i>	<i>GH</i>	<i>AC</i>	<i>E</i>	<i>AH</i>	<i>AG</i>	<i>J</i>	<i>B</i>	<i>C</i>	<i>e</i> ₁	<i>e</i> ₂	<i>y</i>
-	-	-	-	+	+	-	+	+	+	-	-	+	+	-	43.7
+	-	-	-	-	+	+	-	+	+	+	-	-	-	+	40.2
-	+	-	-	+	-	+	+	-	+	-	+	-	-	+	42.4
+	+	-	-	-	-	-	-	-	+	+	+	+	+	-	44.7
-	-	+	-	-	-	+	+	+	-	+	+	-	+	-	42.4
+	-	+	-	+	-	-	-	+	-	-	+	+	-	+	45.9
-	+	+	-	-	+	-	+	-	-	+	-	+	-	+	42.2
+	+	+	-	+	+	+	-	-	-	-	-	-	+	-	40.6
-	-	-	+	+	+	-	-	-	-	+	+	-	+	+	42.4
+	-	-	+	-	+	+	+	-	-	-	+	+	-	-	45.5
-	+	-	+	+	-	+	-	+	-	+	-	+	-	-	43.6
+	+	-	+	-	-	-	+	+	-	-	-	-	+	+	40.6
-	-	+	+	-	-	+	-	-	+	-	-	+	+	+	44.0
+	-	+	+	+	-	-	+	-	+	+	-	-	-	-	40.2
-	+	+	+	-	+	-	-	+	+	-	+	-	-	-	42.5
+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	46.5

- Fit a model to the data including only the effects that appear significant on the normal plot, and calculate the residuals from the fitted model.
- Calculate the sample variance of residuals from the last model for the - and + level of each column in Table 12.24, and calculate the dispersion statistic (given by Equation (12.5)) for each column. Make a normal plot of the dispersion statistics and identify any potential dispersion effects.
- Use the `lme` function in the `nlme` package to simultaneously estimate the location-dispersion effect using the method of iterative weighted least squares.
- What factor levels do you recommend to maximize the tensile strength with minimum variation?